MODEL FOR THE DEPOSITION OF A LIQUID FILM ONTO A SOLID SURFACE

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We consider the deposition of a film onto a thin tape drawn from a liquid bath. We obtain the dependence of the thickness of the film of liquid on the physical and regime parameters.

The deposition of liquid films of controlled thickness onto solid substrates is of interest in many applications. We mention, in particular, the preparation of metalloceramic materials and the deposition of electrically insulating films on metal surfaces, which is important in the manufacture of electrotechnical steel [1]. In the contemporary technology, a metal tape is passed through a bath filled with a homogeneous liquid, colloid solution, or a finely dispersed suspension of the appropriate composition. The tape is wetted by the liquid, passes through the pressing rollers (Fig. 1a), and is then dried. The thickness of the film affects the effective electrical insulating properties of the magnetic circuit coils from electrotechnical steel and the specific losses and other important operational characteristics of metalloceramic materials [2]. The program for improving the quality of electrotechnical steel envisages an automatic site for the deposition of the liquid film. This requires the development of a physical model for the process which would predict the dependence of the film thickness on the velocity with which the tape is drawn, on the pressing force of the rollers, on the radius and angular velocity of the rollers, on the pressure drop between the bath and the surrounding medium, and on the physical properties of the liquid.

Since the radius of the rollers $R \sim 0.1$ m and the characteristic thickness of the film $h_\infty \sim 10^{-6}$–10^{-4} m it is natural to construct the model using the known hydrodynamic approximation of a thin lubricating layer [3]. We introduce Cartesian coordinates with origin in the plane of the drawn substrate at a point which corresponds to the minimum gap between the rollers (Fig. 1b). Near this point $x = (R + h_0)\tan\varphi$, $h = h_0 + R(1 - \cos\varphi)$. Since $h \ll R$, it hence follows, approximately, that

$$x \approx R\varphi,\ h \approx h_0(1 + R\varphi^2/2) \approx h_0(1+\xi^2),\qquad \xi = x/\beta,\quad \beta^2 = 2h_0R, \tag{1}$$

and in the gap (small φ), $dh/dx \approx \varphi \ll 1$, which justifies the use of this approximation. The Navier–Stokes equation in the case under consideration can be considerably simplified:

$$\mu\frac{\partial^2 v}{\partial y^2} = \frac{\partial p}{\partial x},\quad 0 = \frac{\partial p}{\partial y}. \tag{2}$$

Neglecting the surface tension effects, the boundary conditions are as follows:

$$v = U,\ y = 0;\ v \approx \Omega R,\ y = h;\qquad p \to p_1,\ x \to -\infty;\ p = p_2,\ dp/dx = 0,\quad x = x_*, \tag{3}$$

where the value x_* of the longitudinal coordinate corresponds to the point where the liquid separates from the surface of the roller (Fig. 1b).

For the first two conditions in (3), the solution of (2) has the form

$$v = -\frac{h^2}{2\mu}\cdot\frac{dp}{dx}\left(1 - \frac{y}{h}\right)\frac{y}{h} + U\left(1 - \frac{y}{h}\right) + \Omega R\frac{y}{h}. \tag{4}$$

The total volume flowrate drawn with the substrate is

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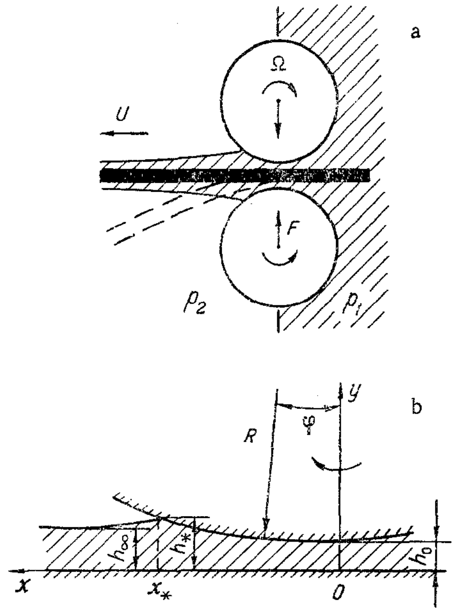

Fig. 1. A diagram of drawing a solid tape from the liquid bath (a) and a section through the liquid layer (b). The dashed regions are occupied by the liquid.

$$Q = \int_0^h v\,dy = -\frac{h^3}{12\mu}\frac{dp}{dx} + \frac{h}{2}(U + \Omega R). \tag{5}$$

In view of the continuity of the flow and the assumed incompressibility of the liquid, Q = const. Writing (5) in the point $x = x_*$ and using the last condition from (3), we obtain

$$Q = \frac{h_*}{2}(U + \Omega R),\ h_* = h_0(1 + z^2),\ z = \frac{x_*}{\beta}. \tag{6}$$

Solving (5) which is considered as a differential equation for the pressure, introducing the dimensionless longitudinal coordinate ξ according to (1), and using the condition on the pressure at $x \to -\infty$ in (3) we obtain with the use of relation (6)

$$\begin{gathered} p = p_1 - \frac{3\beta\mu}{2h_0^2}(U + \Omega R)\left[\varphi_1(\xi) - \frac{h_*}{h_0}\varphi_2(\xi)\right], \\ \varphi_1(\xi) = \frac{1}{2}\left(\frac{\xi}{1+\xi^2} + \frac{\pi}{2} + \operatorname{arctg}\xi\right), \\ \varphi_2(\xi) = \frac{3}{8}\left[\left(1 + \frac{2/3}{1+\xi^2}\right)\frac{\xi}{1+\xi^2} + \frac{\pi}{2} + \operatorname{arctg}\xi\right]. \end{gathered} \tag{7}$$

The minimum thickness of the layer h_0, which is the remaining unknown, can be expressed in terms of z and other parameters from the condition on pressure at $x = x_*$ in (3), i.e., from the following equation which follows from (7)

$$\begin{gathered} a\eta^{3/2} = f_1(z),\quad \eta = \frac{h_0}{R},\quad a = \frac{\sqrt{2}}{3}\frac{R(p_1 - p_2)}{\mu(U + \Omega R)}, \\ f_1(z) = -\frac{1 - 3z^2}{2}\left(\frac{\pi}{2} + \operatorname{arctg} z\right) + \left(\frac{3}{2} - \frac{1}{1+z^2}\right)z. \end{gathered} \tag{8}$$

The second equation which is necessary for the determination of the quantity z (i.e., the point of separation of the liquid from the surface of the roller) will be found from the requirement that the total disjoining pressure on the rollers by the liquid layers (which tends to separate the rollers) is equal in modulus to the externally applied force F which pushes the rollers together. In the approximation of a thin lubricating layer for the situation shown in Fig. 1a, we obtain the equation

$$F = \beta (p_1 - p_2) z + \beta \int_{-\infty}^{z} (p - p_1)\, d\xi, \tag{9}$$

where $p = p(\xi)$ is determined in (7). The integral in (9) will be calculated by replacing the lower limit of integration by $-\xi$, and letting ξ tend to infinity. As a result, we obtain

$$\beta \int_{-\infty}^{z} (p - p_1)\, d\xi = \frac{3\beta^2 \mu}{h_0^2} (U + \Omega R) \left[1 + \frac{z(1 - z^2)}{2} \left(\frac{\pi}{2} + \operatorname{arctg} z \right) - \frac{3z^2}{2} \right]$$

and, furthermore, by expressing $p_1 - p_2$ in terms of z and other quantities using (8) we obtain the required equation

$$b\eta = f_2(z),\ b = \frac{F}{3\mu (U + \Omega R)},\ f_2(z) = \frac{1 - z^2}{1 + z^2}, \tag{10}$$

which supplements (8).

Solving Eqs. (8) and (10), we have

$$\eta = \frac{f_2(z)}{b},\ f(z) = \frac{f_1(z)}{f_2^{3/2}(z)} = c = \sqrt{6}\, \frac{(p_1 - p_2)[\mu (U + \Omega R)]^{1/2}}{F^{3/2}}. \tag{11}$$

The dependences $f_1(z)$, $f_2(z)$ and $f(z)$ are shown in Fig. 2. It is not difficult to see that there are several principally different stationary regimes of how the process can be carried out. We consider first the regimes with a positive pressing force $F > 0$ which corresponds to $f_2(z) > 0$, i.e., $|z| < 1$. The situation when the pressure p_1 in the liquid film exceeds the pressure p_2 in the surrounding medium corresponds to $f_1(z) > 0$, i.e., $0.47 < z < 1$. The interval $-1 < z < 0.47$ corresponds to the regime with counterpressure when $p_1 < p_2$. A decrease of the parameter c in (11) from a large positive quantity leads to a decrease of z, i.e., to an accelerated separation of the liquid from the surface of the roller, as can be seen from the analysis of the dependence f(z) in Fig. 2. This acceleration takes place slowly at first and then, in the regime with counterpressure, it becomes very sharp, practically discontinuous. In the region $z > 0$, this assists the decrease of the asymptotic thickness of the liquid film which is reached for $x \to \infty$:

$$h_\infty = \frac{Q}{U} = \frac{(1 + z^2) h_0}{2} \frac{U + \Omega R}{U} \tag{12}$$

(here, we used (6)). In the region $z < 0$, on the other hand, this leads to an increase of h_∞. The character of change of the minimum thickness h_0 of the gap depends on the manner in which this decrease of c takes place, i.e., which parameters are changed to bring about this decrease. For example, if the dimensionless parameter b in (10) remains constant or changes weakly then h_0 (which is proportional to the function $f_2(z)$ in Fig. 2) increases at first, reaches a maximum near $z = 0$, and then begins to decrease. It is clear that the increase of h_0 assists the increase of the quantity h_∞ from (12). Therefore, the dependences of h_∞ on different physical and regime parameters are, in general, nonmonotonic and fairly complicated. They can be constructed by using (11), (12) and Fig. 2.

The negative pressing force $F < 0$ is, in steady-state conditions, possible in situations when the liquid layer manifests "suction" as a result of rarefaction in the liquid layer. This is characteristic for the regimes with counterpressure. In this case, the pressure in the liquid layer which satisfies the obvious inequality $p_2 > p > p_1$ does not compensate the external pressure p_2. In addition, the regime with negative F can be established also for $p_1 - p_2 > 0$, if the liquid layer separates from the roller at negative x.

In realistic conditions, the inequality $b \gg 1$ holds. This is also necessary for the validity of the approximation of thin lubricating layer. In addition, c is usually a positive quantity which is small relative to unity. In this case, one can approximately take $z \simeq 0.5$ and, in accordance with (8) and (10), $h_0 \approx 0.6\, R/b$. We then have from (12)

$$h_\infty \approx 1{,}125 \frac{\mu R (U + \Omega R)^2}{UF}, \tag{13}$$

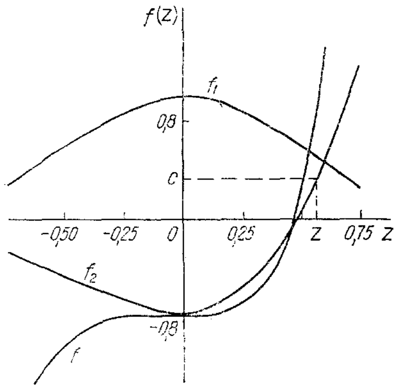

Fig. 2. Functions $f_1(z)$, $f_2(z)$ and $f(z)$ and the scheme for the determination of z from given c.

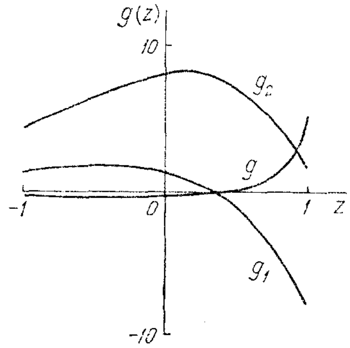

Fig. 3. Functions $g_1(z)$, $g_2(z)$ and $g(z)$.

i.e., the thickness of the deposited film increases linearly with increasing viscosity of the liquid and increasing radius of the rollers and decreases in proportion to the pressing force. The quantity h_∞ as a function of U for a constant Ω reaches maximum for $U = \Omega R$.

The preferred angular velocity of the roller rotation can be established either by the application of external moments of forces, or by setting up frictional devices with given characteristics. Therefore, it is of interest to calculate the moment of resistive forces acting on the rollers as a result of the thin liquid layer. In accordance with (4)-(6), the tangential viscous tension at the surface of the roller which is adjacent to the layer can be represented in the form

$$\tau = -\mu \frac{\partial v}{\partial y}\bigg|_{y=h} = -\frac{h}{2}\frac{dp}{dx} + \frac{\mu}{h}(U - \Omega R) =$$
$$= \frac{3\mu}{h}\left(\frac{h_*}{h} - 1\right)(U + \Omega R) + \frac{\mu}{h}(U - \Omega R),$$

where h is a function of ξ determined in (1). Hence, we obtain in the approximation of thin lubricating layer

$$M = \beta R \int_{-\infty}^{z} \tau d\xi = -\mu R\left(\frac{h_0 R}{2}\right)^{1/2}[g_1(z)U + g_2(z)\Omega R],$$
$$g_1(z) = (1 - 3z^2)(\pi/2 + \operatorname{arctg} z) - 3z,$$
$$g_2(z) = (5 - 3z^2)(\pi/2 + \operatorname{arctg} z) - 3z \tag{14}$$

(the positive direction of rotation of the rollers is taken clockwise, as shown in Fig. 1b). If the external moment of forces is zero and one can neglect friction (with the exception of the viscous friction in the liquid layer) the following angular velocity of roller rotation is established as the tape is drawn:

$$\Omega = g(z)(U/R), \; g(z) = -g_1(z)/g_2(z). \tag{15}$$

The dependences $g_1(z)$, $g_2(z)$ and $g(z)$ are shown in Fig. 3. It is seen that there are many instances when the roller is not dragged by the tape in the direction of its motion (i.e., clockwise rotation, see Fig. 1b). For $F > 0$ ($|z| < 1$), the clockwise rotation takes place only when $z > 0.4$. In the opposite case, the roller begins to rotate counterclockwise and the linear velocity on the surface facing the drawn tape is opposite to the velocity of the tape. This result which seems paradoxical is, in fact, fully understandable. This phenomenon of reverse rotation takes place only in regimes with counterpressure. In this case, the viscous pressure caused by the Poiseuille component of the total flow in the thin layer which rotates the roller counterclockwise can exceed, in absolute value, the pressure from the Couette component due to drawing of the tape which rotates the roller clockwise. A visual

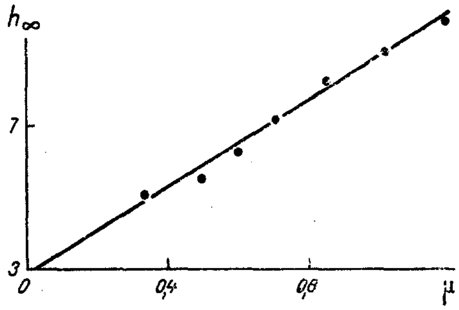

Fig. 4. Thickness of the liquid film as a function of the viscosity of the liquid. The points are experimental and the straight line is theoretical.

analogous phenomenon of reverse rotation takes place also when a cylinder is rotated by a liquid jet of finite width which falls on the cylinder normally to the generating line [4].

Although no special experiments have been constructed to verify the developed model, an analysis of the experimental and industrial data indicates that the model is, on the whole, adequate. The dependence of the width of the film on the various parameters follows the model ideas which were developed here.

By way of example, Fig. 4 shows the data concerning the dependence h_∞ on the viscosity of the liquid obtained during the deposition of an electrically insulating film of magnesium phosphate on a steel tape. The liquid is a dispersion of small magnesium oxide particles in water, with the addition of orthophosphoric acid. Depending on the concentration, the viscosity of the dispersion varies from 0.32 to 1.2 P. It is seen that the experimental points lie close to the straight line which follows from the theory.

In conclusion, we note that, in the future, it is expedient to expand the theory to include the effects of surface tension and the wetting of the solid surface by the liquid. In addition, it is of interest to generalize theory to accommodate a drawing process which is accompanied by the bending of the tape on the rollers, as shown by the dashed line in Fig. 1a. Finally, in the applied design, it is important to analyze boundary phenomena which take place near the lateral sides of the drawn film. To this end, it is necessary to consider a nonplanar problem of the hydrodynamics of a thin liquid layer.

NOTATION

F, pressing force of the rollers; h, thickness of the liquid film; h_0, h_*, h_∞, minimum thickness, the thickness which corresponds to the separation of the film from the roller, and the asymptotic value of the thickness, respectively; M, moment of the resistance forces: p, pressure; p_1 and p_2, pressures inside and outside the liquid bath, respectively; Q, volume flow rate of the liquid; R, radius of the roller; U, velocity with which the solid tape is drawn; v, velocity of the liquid; x, y, Cartesian coordinates; z, critical value of ξ which corresponds to the separation of liquid from the surface of the roller; β, linear scale in (1); ξ, dimensionless longitudinal coordinate; η, dimensionless minimum thickness of the liquid film; μ, viscosity of the liquid; and Ω, angular velocity of the rollers.

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